The optimum solution with $\tau = 0.403$, however, calls for a sail lift coefficient of $2 \cdot 896 \cdot 0.403/362 = 1.99$, likely unattainable with conventional sails within this sail plan. Assuming $C_{L_{max}} = 1.7$ leads to $\tau_{max} = 1.7 \cdot 362/2 \cdot 896 = 0.343$; and using starting values for (x_o, β_o) of (1.884, 0.445), the iteration (22) converges in 3 steps to the values (1.936, 0.441), giving $y = 0.602$, about 1% lower than the unconstrained optimum. This solution occurs at $\gamma = 47.7°$ from the true wind (Eq. 12).

VII. Conclusions

A significant range of operating conditions has been identified in which hull and sail characteristics are unaffected by wavemaking and heeling. In this regime, boat speed is directly proportional to true wind speed, and the vehicle can be described in an analytic model by three or four dimensionless parameters. Optimum sail trim is found to depend on hull parameters, as well as on characteristics of the sail, and vessel course and speed.

Conspicuous in their absence are considerations of lift-curve-slopes and angles of attack of sails or hull; these are found to have no direct bearing on performance, within the assumptions of the model. Lift-to-drag ratios of hull and sail do not appear as fundamental parameters, as is often assumed; and maximum lift-to-drag ratios are in general not optimum operating points for either hull or sails. An optimum proportion between hull and sails, as represented by the hull-to-sail capture ratio κ, is located, but reaching this optimum would require much larger sails than are ordinarily used.

Although the paper has been limited to a special range of operation in which simple force equations suffice, much of the present analytic approach and method of solution clearly can apply to a broader analysis of sailing performance. In particular, the treatment of sail forces and trim developed here is the first analytic alternative to use of the *Gimcrack* coefficients or similar empirical data in predicting sailing speeds from either model tests or theories of sailing hull performance.

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